Exercise 58

Find a formula for a function that has vertical asymptotes $x = 1$ and $x = 3$ and horizontal asymptote $y = 1$.

Solution

To have a vertical asymptote at $x = 1$, place a factor of $x - 1$ in the denominator. To have a vertical asymptote at $x = 3$, place a factor of $x - 3$ in the denominator. To have a horizontal asymptote at $y = 1$, place x^2 in the numerator so that the limit of the function as $x \to \infty$ is 1.

$$f(x) = \frac{x^2}{(x-1)(x-3)}$$